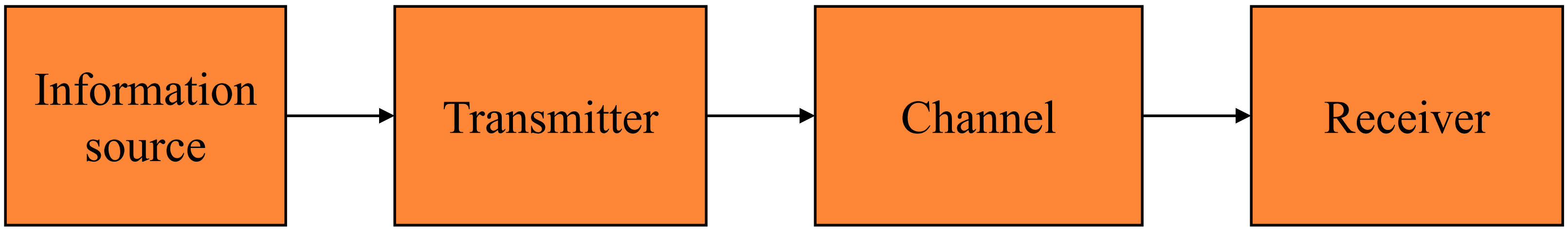
COMMUNICATION ENGINEERING

Introduction to Information & Entropy

INTRODUCTION

- What is communication



A communication system

- Process of conveying message or information
- Deals with the flow of information bearing signal from one place to another over a communication channel
- Information can be electrical signals, words, pictures etc.

INTRODUCTION

- What is Information and how to define the measure of an amount of information?

- How can it be applied to improve the communication of information?

To answer these questions we need to study **Information theory**

INFORMATION THEORY

Information theory deals with the problem of efficient and reliable transmission of information

INTRODUCTION

- Information theory is a branch of probability theory which may be applied to the study of communication system

- It deals with mathematical modelling and analysis of a communication system

- It also answers the fundamental questions in communication
  - the irreducible complexity below which signal cannot be compressed
  - ultimate transmission rate for reliable communication over a noisy channel

WHAT IS INFORMATION

- Information is knowledge that can be used
- The amount of information associated with various messages are different. Some messages contain more information while others contain less information
- For eg. If a dog bites a man : its no news
  but if a man bites a dog : it's a news
- Thus MORE the probability of an event LESSER the information it contains
- Amount of information depends upon the uncertainty of the event rather than its actual content.

INFORMATION SOURCES

- Device which produces messages either analog or discrete, the outcome of which is selected at random according to the probability distribution
- Discrete source has finite set of symbols as outputs
  - Set of symbols---SOURCE ALPHABET
  - Elements of set--- SYMBOLS OR LETTERS
- Information Source can be classified as
  - Having memory
  - Being memory less
- We will study Discrete Memory less Source (DMS)

INFORMATION CONTENT OF A SYMBOL

- Mathematical measure of information should be
  - proportional to the uncertainty of the outcome
  - Information content in independent outcomes should add

- Let a DMS denoted by X having alphabet { $x_{1,}\ x_{2,\ldots\ldots}\ x_m$ } probability P(x_j) is the probability of occurrence of symbol x_i and the amount of information be I(x_i)

$$I(x_i) = \log_b (1/ P(x_i)) = - \log_b P(x_i)$$

Unit of $I(x_i)$

- The unit of $I(x_i)$ is
  - bit if b=2
  - nat if b=e
  - hartlley/decit if b=10
- It is standard to use b=2

$$\log_2 a = \log_e a / \log_e 2$$

Properties of $I(x_I)$

- $I(x_i) = 0$ for $P(x_i) = 1$
- $I(x_i) \geq 0$
- $I(x_i) \geq I(x_j)$ if $P(x_i) \leq P(x_j)$
- $I(x_i, x_j) = I(x_i) + I(x_j)$

  if x_i and x_j are independent

PROBLEMS

1. In a binary PCM '0' occurs with probability of ¼ and '1' occurs with probability of ¾ .Calculate the amount of information carried by each bit.

2. If there are M equally likely and independent symbols , what is the amount of information carried by each symbol .Given $M=2^N$ and n is an integer.

ENTROPY (AVERAGE INFORMATION)

- In a practical communication system we transmit a long sequence of symbols from an information source

- As flow of information can fluctuate because of randomness involved in the selection of symbols, we require average information content of the symbols in a long message

- Average information content per source symbol is called ENTROPY of the source.

- Entropy is a measure of uncertainty

Mathematical Expression for Entropy

- The mean value of $I(x_i)$ over the alphabet of source X with m different symbols is given by

$$H(X) = \sum_{i=1}^{m} P(x_i)\, I(x_i) = -\sum_{i=1}^{m} P(x_i) \log_2 P(x_i)$$

Entropy of binary source

- For a binary source m=2
- Entropy is

$$H(X) = -\sum_{i=1}^{2} P(x_i)\log_2 P(x_i)$$

Let $P(x_1) = p$ and $P(x_2) = (1-p)$

$$H(X) = -P(x_1)\log_2 P(x_1) - P(x_2)\log_2 P(x_2)$$

$$= -p\log_2 p - (1-p)\log_2 (1-p)$$

The condition for maximum entropy is given by differentiating H(X) w.r.t p and equate it to zero

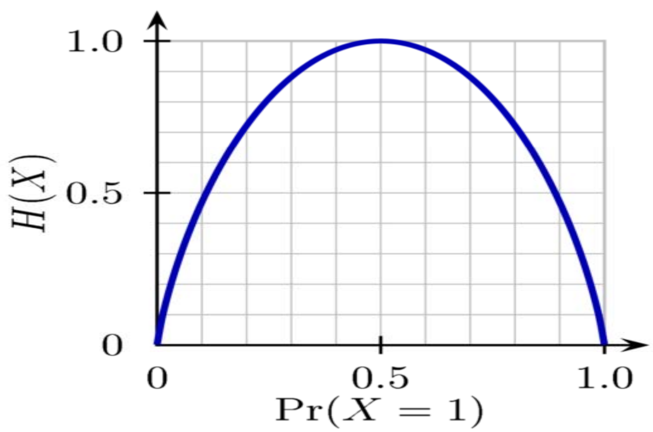
By putting

$$d/dp\ (H(x))=0$$

we get P=1/2 i.e both messages are equally likely

CONTD…

- A plot of H as a function of p



- The maximum value of H at p=1/2 is

$$H_{max} = 1/2 \log 2 + 1/2 \log 2 = 1 \text{ bit/message}$$

CONTD…

- Similarly for M ary case entropy is maximum when all messages are equally likely

$$H(X) = \sum_{i=1}^{m} P(x_i)\, I(x_i) = -\sum_{i=1}^{m} P(x_i) \log_2 P(x_i)$$

Probability of each symbol= 1/M

$$H(X) = \sum_{i=1}^{m} \left(\frac{1}{M}\right) \log_2 \left(\frac{1}{\frac{1}{M}}\right) = \sum_{i=1}^{m} \left(\frac{1}{M}\right) \log_2 (M)$$

$H_{max} = \log M$ bit/message

PROBLEMS

1. Consider a source X which produces five symbols with probabilities 1/2,1/4,1/8, 1/16 and 1/16.Find the source entropy H(X).

2. Prove that $0 \leq H(X) \leq \log_2 m$ where m is the size of the alphabet of X.

THANK YOU